VIRTUAL MASSES COEFFICIENTS AND AERODYNAMIC DAMPING CONSTANT OF VIBRATING CIRCULAR ARRAYS OF THIN PROFILES

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In the study of some questions in aeroelasticity it is necessary to determine the virtual masses and coefficients of aerodynamic damping for bodies undergoing small oscillations. In regions of arrays, simulating the blades of working wheels and directing apparatus of turbomachines, this problem has been studied only for oscillations of arrays in an incompressible liquid. In [1] the virtual masses of a straight array of plates were determined, and in [2] the virtual masses of a ring-shaped array of blades, used for modeling the steps of axial machines, were determined. In these works, in particular, the effect of the interaction between blades undergoing small harmonic oscillation with a constant phase shift on the coefficients of virtual masses was studied.

In a compressible liquid such problems have been studied only for separate bodies. However, in studying the oscillations of arrays in a gas, aside from the question of the interaction of the blades, it is of special interest to investigate regimes near resonance, which arise when the real part of the characteristic frequency of the oscillations of the gas near the array equals the frequency of oscillations of the blades.

In this work the virtual masses and coefficients of damping for an immobile circular array of thin profiles undergoing small harmonic oscillations with a constant phase shift in an ideal compressible liquid are determined. Such arrays are usually used to model blade diffusers and directing apparatus of centrifugal turbomachines. The calculations performed showed that in regimes near resonance two types of phenomena are observed: resonance with a sharp increase in the amplitude of the complex coefficients of the forces acting on the profiles in the array and resonance absorption, in which the amplitude of these coefficients decreases while the phase changes sign.

1. We shall study the problem of the propagation of small disturbances, emitted by an immobile circular array, whose profiles undergo small oscillations according to the same harmonic law with a constant phase shift $\mu = 2\pi m/N$ ($m = 0, 1, \ldots, N-1$; N the number of profiles in the array), in an ideal gas. We shall assume that the profiles are infinitely thin, weakly curved arcs. We denote by R_1 and R_2 the inner and outer radii of the array (Fig. 1). We introduce the dimensionless polar coordinates ρ/R_1 and θ, placing the origin of the coordinate system at the center of the array.

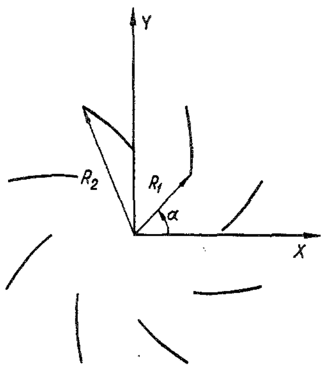

Fig. 1

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Under these assumptions the oscillations of the n-th profile of the array can be described in the form

$$w^{(n)}(s) = R_1 f(s) \exp\,[i(n\mu - \omega t)],$$

where $w^{(n)}$ is the displacement of the point of the n-th profile along the normal; ω is the circular frequency of the oscillations; f(s) is a dimensionless complex function determining the form of the oscillations and, s is the coordinate of points along the arc of the profile, measured from the inner edge of the fixed central position of the profile L_n.

Assuming that the perturbed motion of the gas is steady we shall seek the velocity potential in the form $\Phi(\rho, \theta, t) = iR_1^2\omega\varphi(\rho, \theta)\times\exp(-i\omega t)$. In the linear approximation the dimensionless complex function $\varphi(\rho, \theta)$, determining the amplitude of the steady state oscillations of the gas, satisfies the homogeneous Helmholtz equation in the entire plane outside the profiles of the array $L = \bigcup\limits_{n=0}^{N-1} L_n$

$$\Delta\varphi + k^2\varphi = 0 \tag{1.1}$$

($k = \omega R_1/a_\infty$ is the wave number and a_∞ is the velocity of sound in the undisturbed gas, with the following conditions:

the profiles of the array are impermeable

$$\left.\frac{\partial\varphi}{\partial\nu}\right|_{L_n} = -f(s)\,e^{in\mu}, \quad n = 0, 1, \ldots, N-1 \tag{1.2}$$

(ν is the normal to the profile);

the radiaton condition, which we write in one of the equivalent forms:

$$\varphi(\rho, \theta) = \sum_{s=-\infty}^{\infty} a_s H_s^{(1)}(k\rho)\,e^{is\theta} \quad \text{for} \quad \rho > R_2 \tag{1.3}$$

($H_s^{(1)}$ is the Hankel function of the first kind);

generalized periodicity

$$\varphi(\rho, \theta + \alpha) = \varphi(\rho, \theta)\exp(i\mu) \tag{1.4}$$

($\alpha = 2\pi/N$ is the spacing of the array along θ); and,

the circulation of the velocity vector around each profile of the array vanishes:

$$\Gamma^{(n)} = 0, \; n = 0, 1, \ldots, N-1 \tag{1.5}$$

($\Gamma^{(n)}$ is the circulation of the velocity around the n-th profile).

2. We shall derive the integral equation that is equivalent to the boundary-value problem (1.1)-(1.5). With the help of the second Green's formula, using (1.3), we write the integral representation of the solution of Eq. (1.1) at any point in the plane, excluding some neighborhood Ω of the profiles of the array L:

$$\varphi(\mathbf{x}) = \int\limits_{\partial\Omega} \left\{ g(\mathbf{y}, \mathbf{x})\frac{\partial\varphi(\mathbf{y})}{\partial\nu_y} - \varphi(\mathbf{y})\frac{\partial g(\mathbf{y}, \mathbf{x})}{\partial\nu_y} \right\} dS_y, \; \mathbf{y} \in \partial\Omega, \tag{2.1}$$

where ν_y is the normal to $\partial\Omega$ at the point $\mathbf{y}$; $g(\mathbf{y}, \mathbf{x}) = H_0^{(1)}(k|\mathbf{y} - \mathbf{x}|)/4i$ is the fundamenetal solution of Eq. (1.1). After differentiating (2.1) we obtain an integral representation for the amplitude of the velocity of small disturbances $\mathbf{v}$, which with the help of the relations from ([3] and (1.1) can be transformed as follows:

$$\mathbf{v} \equiv \nabla_x\varphi = -\int\limits_{\partial\Omega} [(\mathbf{v}_y \times \mathbf{v}) \times \nabla_y g + (\mathbf{v}_y \cdot \mathbf{v})\,\nabla_y g + k^2 g\varphi\mathbf{v}_y]\,dS_y. \tag{2.2}$$

On transferring from $\partial\Omega$ to infinitely thin profiles of the array L the form of (2.2) remains the same, if the integration is performed on one side of the profile and φ and $\mathbf{v}$ are replaced by the values of their jumps on crossing L: $\Gamma = \varphi^- - \varphi^+$, $\gamma = (\mathbf{v}^- - \mathbf{v}^+)\cdot\boldsymbol{\tau}$, $\delta = (\mathbf{v}^- - \mathbf{v}^+)\cdot\boldsymbol{\nu}$ (τ is the unit tangent vector to L). Since the integrand in the representation (2.2) does not contain second derivatives of g, in accordance with [4] we find the integrodifferential equation for the unknown functions $\gamma(s)$ and $\Gamma(s)$:

$$\int_L \left\{\gamma\left[(\mathbf{v}_x\cdot\boldsymbol{\tau}_y)\frac{\partial g}{\partial \mathbf{v}_y} - (\mathbf{v}_x\cdot\mathbf{v}_y)\frac{\partial g}{\partial \tau_y}\right] + \Gamma k^2 g\,(\mathbf{v}_x\cdot\mathbf{v}_y)\right\} dS_y = F(\mathbf{x}),\ \ \mathbf{x}\in L, \tag{2.3}$$

where the right side is

$$F(\mathbf{x}) = -\frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial \mathbf{v}_x}\right)^- + \left(\frac{\partial\varphi}{\partial \mathbf{v}_x}\right)^+\right] - \int_L \left[\left(\frac{\partial\varphi}{\partial \mathbf{v}_y}\right)^- - \left(\frac{\partial\varphi}{\partial \mathbf{v}_y}\right)^+\right](\mathbf{v}_y\cdot\nabla_y g)\,dS_y.$$

The functions sought are also related by the relations

$$\partial\Gamma(\mathbf{y})/\partial\tau_y = \gamma(\mathbf{y}),\ \mathbf{y}\in L,\ \int_{L_j}\gamma(\mathbf{y})\,dS_y = 0,\quad j = 0, 1, \ldots, N-1, \tag{2.4}$$

which follow from the definition of these functions and the condition (1.5).

We note that Eqs. (2.3) and (2.4) and the method of solution described below are valid for the general case, when the Neumann boundary-value problem for an arbitrary collection of a finite number of smooth bounded arcs is studied. In this case, by virtue of the conditions (1.2) and (1.4), Eq. (2.3) can be rewritten in the form

$$\int_{L_0}\{\gamma(y)K_0(\mathbf{y},\mathbf{x}) + \Gamma(y)K_1(\mathbf{y},\mathbf{x})\}\,dS_y = f(\mathbf{x}),\ \mathbf{x}\in L_0. \tag{2.5}$$

Here $K_0(\mathbf{y},\mathbf{x}) = (\boldsymbol{\tau}_y\cdot\mathbf{v}_x)\dfrac{\partial g_N^m(\mathbf{y},\mathbf{x})}{\partial \mathbf{v}_y} - (\mathbf{v}_y\cdot\mathbf{v}_x)\dfrac{\partial g_N^m(\mathbf{y},\mathbf{x})}{\partial\tau_y}$; $K_1(\mathbf{y},\mathbf{x}) = k^2 g_N^m(\mathbf{y},\mathbf{x})\times(\mathbf{v}_x\cdot\mathbf{v}_y)$; S_y is the arc coordinate on L_0; $\mathbf{x},\mathbf{y}\in L_0$; $g_N^m(\mathbf{y},\mathbf{x}) = \sum_{j=0}^{N-1} e^{ij\mu}H_0^{(1)}(k|\mathbf{y}_j-\mathbf{x}|)$, $\mathbf{y}_j\in L_j$ is the elementry solution of Eq. (1.1) for a circular array of point singularities with the coordinates $\mathbf{y}_j = (\rho, \theta + j\alpha)$.

3. We shall determine the forces acting on the zeroth profile of the array, whose oscillations are described by

$$w(\rho,\theta,t) = \sum_{m=1}^{N_0} q_m(t)\,f_m(\rho,\theta), \tag{3.1}$$

where $q_m(t)$ are generalized coordinates with the dimension of length; $f_m(\rho, \theta)$ are the forms of the oscillations; and, N_0 is the number of generalized coordinates.

We shall study, for a fixed form of the oscillations, the generalized hydrodynamic force

$$Q_n(t) = \int_{L_0}\Delta p(\rho,\theta,t)\,f_n(\rho,\theta)\,dS_y, \tag{3.2}$$

in which the pressure drop on the profile Δp is calculated in the linear approximation with the help of the Cauchy–Lagrange integral

$$\Delta p(\rho,\theta,t) = -\rho_0\left[\left(\frac{\partial\Phi}{\partial t}\right)^- - \left(\frac{\partial\Phi}{\partial t}\right)^+\right]. \tag{3.3}$$

We introduce the complex coefficients

$$G_{mn} = \mu_{mn} + \frac{i}{\omega}\lambda_{mn} = -R_1\rho_0\int_L \Gamma_m f_n\,dS_y \tag{3.4}$$

($\Gamma_m(s)$ is the solution of Eqs. (2.4) and (2.5) with right side $f(s_x) = f_m(\rho,\theta)$). For harmonic oscillations, described by (3.1), by virtue of the linearity of the problem we rewrite, based on (3.3) and (3.4), the relation (3.2) in the form

$$Q_n(t) = -\sum_{m=1}^{N_0}\mu_{mn}\ddot{q} - \sum_{m=1}^{N_0}\lambda_{mn}\dot{q}_m.$$

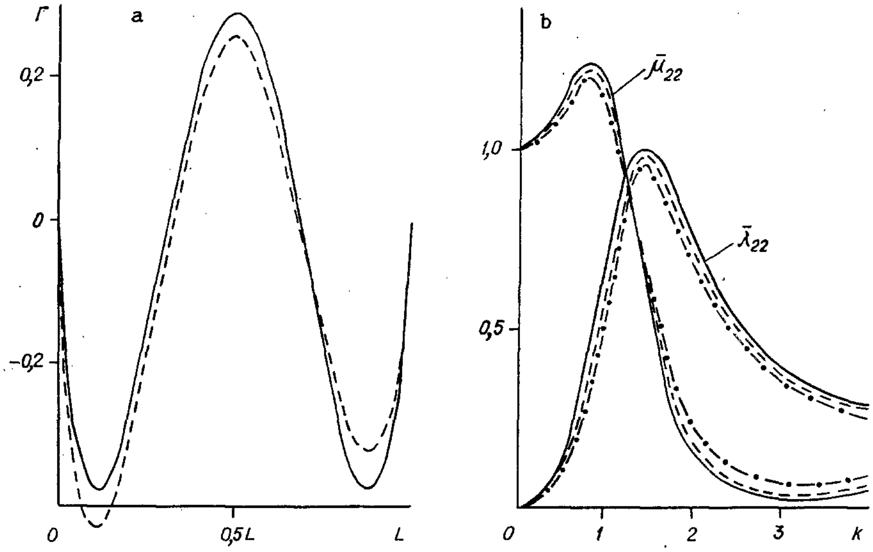

Fig. 2

Here the coefficients μ_{mn} characterize the inertial forces, which have the dimension of mass and are called the coefficients of the virtual msses; λ_{mn} are the damping coefficients; and, $\dot{q}_m$ and $\ddot{q}_m$ are the generalized velocities and accelerations.

Therefore the problem of determining the aerodynamic forces acting on a profile in the array has been reduced to calculating the complex coefficients G_{mn}, which can be expressed in terms of the solution of Eqs. (2.4) and (2.5).

4. The singular introdifferential equation (2.4) has the feature that its kernel has a first-order pole plus a term with a logarithmic singularity. We note that direct numerical methods for solving equations of this type not currently widely employed [5]. It was proposed previously [6] that the method of discrete singularities, which is the simplest method among the direct methods employed for solving singular integral equations with a kernel of the Cauchy type, be employed for numerical investigation of an analogous homogeneous equation. The complex characteristic frequencies of oscillations of the gas, calculated in [6], outside some thin bodies were virtually identical to the frequencies obtained for these regions by the method of splicing. The application of this method to the solution of inhomogeneous equations (2.4) and (2.5), however, made it necessary to improve this method.

In constructing the solution by the method of discrete singularities the test problem for calculating the virtual masses of a thin plate of length $2a$, undergoing translational oscillations, an unsymmetric distribution of the jump in the amplitude of the potential $\Gamma(s)$ along the plate was obtained (Fig. 2a, broken line, the plate was divided into 20 elementary sections, $k = 4.0$). It was established that for a fixed number of discrete singularities on the plate, as k increases the asymmetry of the numerical solution increases and affects slightly the values of the overall characteristics. The coefficients of virtual masses $\bar{\mu}_{22}=\mu_{22}/\rho_0\pi a_\infty^2$ and aerodynamic damping $\bar{\lambda}_{22}=\lambda_{22}/\omega\rho_0\pi a_\infty^2$ (Fig. 2b, broken lines) agree well with the results obtained with the help of Haskind's method [7] (solid lines). The asymmetry of the characteristics distributed along the plates is a consequence of the asymmetry of the matrix of the system of algebraic equations with this type of discretization of Eq. (2.5) owing to the presence of a term with a logarithmic singularity in the kernel of the equation. We note that when the number of discrete singularities increases the asymmetry of the matrix and of the solution decreases.

To avoid this error in the calculations, we shall study a modification of the method of discrete singularities. To this end we divide the profile L_0 uniformly into 2M nonintersecting elementary sections ℓ_m so that $L_0 = \cup_{m=1}^{2M} \ell_m$. We denote by $\mathbf{x}_m$ the points lying between the intervals ℓ_{2m} and ℓ_{2m+1} ($m = 0, \ldots, M$, ℓ_0 and $\ell_{2M+1} = \emptyset$) and by $\mathbf{y}_m$ the points between ℓ_{2m-1} and ℓ_{2m} ($m = 1, \ldots, M$). We introduce on L_0 the system of characteristic functions

$$\Psi_m(s)=\begin{cases}1, & \text{if } s\in l_{2m-1}\cup l_{2m},\\ 0, & \text{if } s\notin l_{2m-1}\cup l_{2m},\end{cases}\quad \Phi_m(s)=\begin{cases}1, & \text{if } s\in l_{2m}\cup l_{2m+1},\\ 0, & \text{if } s\notin l_{2m}\cup l_{2m+1},\end{cases}$$

with whose help we shall write the approximate solutions $\bar{\gamma}(s)$ and $\bar{\Gamma}(s)$ of Eqs. (2.4) and (2.5) as linear combinations

$$\bar{\gamma}(s)=\sum_{m=1}^{M}\bar{\gamma}_m\Psi_m(s)\text{ and }\bar{\Gamma}(s)=\sum_{m=0}^{M}\bar{\Gamma}_m\Phi_m(s) \tag{4.1}$$

($\bar{\gamma}_m$ and $\bar{\Gamma}_m$ are the approximate values of the functions sought at the points of interpolation y_m and x_m, respectively). We introduce the intensity of the jumps in the tangential com-component of the velocity vector on intervals of the profile $\tilde{\gamma}_m=\int_{l_{2m}\cup l_{2m-1}}\gamma(s)\,ds\approx\bar{\gamma}_m\Delta_m$ (Δ_m determines the length of the section $\ell_{2m}\cup\ell_{2m-1}$). Then it follows from (4.1) and (2.4) that the following relations hold:

$$\bar{\Gamma}_n=\sum_{m=1}^{n}\tilde{\gamma}_m,\ \bar{\Gamma}_0=\bar{\Gamma}_M=0,\quad n=1,\ldots,M-1. \tag{4.2}$$

To obtain the discrete analog of Eq. (2.5) we shall approximate the integral operator I_0, whose kernel contains a pole, by analogy to the method of discrete singularities. To this end we place at the points y_m the point singularities of the function K_0 with intensities $\tilde{\gamma}_m$, i.e., $I_0(\mathrm{x})=\sum_{m=1}^{M}\tilde{\gamma}_mK_0(\mathrm{y}_m,\mathrm{x})$. We shall approximate the second integral operator I_1 with the kernel having a logarithmic singularity in accordance with the general scheme of the method of self-regularization [8]. Substituting the approximation (4.1) for the solution $\Gamma(s)$ into the integral operator I_1 we obtain

$$I_1(\mathrm{x})=\sum_{m=1}^{M}\bar{\Gamma}_m\int_{l_{2m}\cup l_{2m+1}}K_1(\mathrm{y},\mathrm{x})\,dS_y.$$

Now choosing the points x_n for the points of colocation we arrive at a system of linear algebraic equations approximating Eq. (2.5),

$$\sum_{m=1}^{M}\left(\tilde{\gamma}_mK_0(\mathrm{y}_m,\mathrm{x}_n)+\bar{\Gamma}_m\int_{l_{2m}\cup l_{2m+1}}K_1(\mathrm{y},\mathrm{x}_n)\,dS_y\right)=f(\mathrm{x}_n), \tag{4.3}$$

the unknown coefficients of which $\tilde{\gamma}_m$ and $\bar{\Gamma}_m$ are related with one another by the additional relations (4.2), n = 1, ..., M − 1. For this choice of points of colocation the principal diagonal of the matrix $\|A_{mn}\|$ with the elements $A_{mn}=\int_{l_{2m}\cup l_{2m+1}}K_1(\mathrm{y},\mathrm{x}_n)dS_y$ is predominant, and for a uniformly divided thin plate it is symmetric. This is achieved owing to the fact that the colocation points are also the interpolation points in the approximation of the integral operator I_1 of Eq. (2.5), whose kernel contains a logarithmic singularity.

The algorithm for calculating the matrix elements A_{mn} is not related directly with the approximation of the solution sought. Any standard numerical integration algorithms can be used to calculate the matrix elements. Taking into account the degree of accuracy of the approximation of the first integral operator, however, in order not to complicate the algorithm for solving Eq. (2.5), we shall confine our attention to the approximation

$$A_{mn}=K_1(\mathrm{x}_m,\mathrm{x}_n)\Delta_m\ \text{ for }\ m\neq n \tag{4.4}$$

TABLE 1

M	Re G_{22}	Im G_{22}	M	Re G_{22}	Im G_{22}
10	1,0860	0,55409	40	1,1584	0,65705
20	1,1348	0,62124	50	1,1629	0,66439
30	1,1506	0,64494			

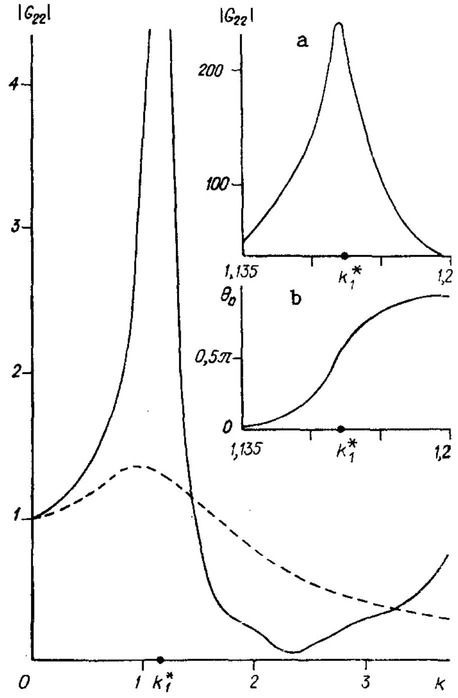

Fig. 3

(Δ_m is the length of the section $\ell_{2m} \cup \ell_{2m+1}$). The diagonal matrix elements A_{nn} contain a logarithmic singularity, which in determining the matrix elements was separated in the form of the term

$$A_{nn} = \int\limits_{l_{2n} \cup l_{2n+1}} K_1(\mathbf{y}, \mathbf{x}_n)\, dS_y = - \int\limits_{l_{2n} \cup l_{2n+1}} \ln |\mathbf{y} - \mathbf{x}_n|\, dS_y + \int\limits_{l_{2n} \cup l_{2n+1}} \widetilde{K}_1(\mathbf{y}, \mathbf{x}_n)\, dS_y,$$

where the first integral is calculated explicitly and the approximation (4.4) ws used for the second integral. To further simplify the computational scheme we shall write the system (4.3), based on (4.2), for the unknowns $\overline{\Gamma}_m$:

$$\sum_{m=1}^{M-1} \overline{\Gamma}_m (K_0(\mathbf{y}_m, \mathbf{x}_n) - K_0(\mathbf{y}_{m+1}, \mathbf{x}_n) + A_{mn}) = f(\mathbf{x}_n). \tag{4.5}$$

The system (4.5) can be regarded as the discrete analog of Eqs. (2.4) and (2.5). We note that the algorithm for calculating the matrix elements using the proposed scheme, unlike the method of discrete singularities, is more complicated only when the diagonal elements A_{nn} are found. However in the case when this method of discretization is employed, the asymmetry in the calculation of the characteristics distributed along the plate is eliminated (Fig. 2a, solid line), and the method is identical to Haskind's method for integral characteristics also (Fig. 2b, broken line).

Numerical studies of the rate of convergence of the proposed computational scheme were performed. The results of a comparison are given in Table 1; analysis of the results shows that the modified scheme is more efficient.

5. In determining the nonstationary aerodynamic forces acting on the vibrating blades of a circular array in a compressible liquid it is of special interest to study the behavior of the forces in the near-resonance regimes. To this end calculations of the virtual masses and damping coefficients were performed for a circular array consisting of twenty radial plates with $R_1 = 1.0$ and $R_2 = 1.5$ for values of k close to the values of the real part k_1^* and k_2^*, which are the first characteristic frequencies of oscillations of the gas near the array with phase shifts between the oscillations of neighboring plates in the array $\mu_1 = \pi$ and $\mu_2 = 0$, respectively [6]. It was established, based on the calculations, that for oscillations of the profiles at a frequency close to the frequency of the characteristic oscillations, resonance phenomena of two types arise. In the first case, when the characteristic oscillations of the gas are localized in the regions of the channels between the blades ($\mu_1 = \pi$) resonance is observed with a sharp increase in the amplitude of the coefficients

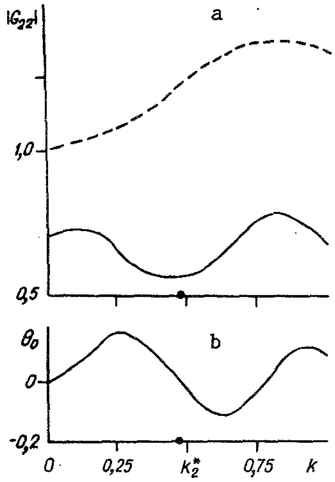

Fig. 4

G_{22} of the aerodynamic forces (Fig. 3a). For comparison Figs. 3 and 4a show (broken line) the values of the amplitude of the coefficients G_{22} for a single plate. We note that on passing through the resonance frequency the phase θ_0 between the oscillations of the plate and the force acting on it changes from 0 to π (Fig. 3b). These results correspond to the results obtained in the study of the behavior of the coefficients of aerodynamic forces in near-resonance regimes of straight arrays of thin plates in a flow [9].

For the phase shift $\mu_2 = 0$ the characteristic oscillations are localized in the interior region of the array ($\rho < R_1$). In this case resonance absorption, with which the amplitude of the coefficients G_{22} decreases (Fig. 4a) while the phase θ_0 changes sign (Fig. 4b), is observed. The question of the existence of absorption resonances was not previously studied in the theory of arrays. In the one-dimensional theory of resonators attached to transmission lines, however, resonance phenomena of both types, analysis of which permits establishing the relation between them, are encountered [10]. Indeed for $\mu_1 = \pi$ the interblade channels can be interpreted as pass-through resonators, connecting the space in front and behind the array. In wave transmission lines the existence of pass-through resonators leads to a sharp increase in the amplitude of oscillations at the resonant frequency. For circular arrays with $\mu_2 = 0$ the interblade channels can be interpreted as wave guides to which a resonator, determined by the internal rgion of the array, is attached. When the wave guide is determined by a resonator, however, as is well known [10], resonance absorption characterized by the same properties as resonance absorption in a circular array is observed in the transmission line.

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